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QCD/String holographic mapping and glueball mass spectrum

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Abstract. Recently Polchinski and Strassler reproduced the high energy QCD scaling at fixed angles from a gauge string duality inspired by the AdS/CFT correspondence. In their approach a confining gauge theory is taken as approximately dual to an AdS space with an IR cut-off. Considering such an approximation (AdS slice) we found a one to one holographic mapping between bulk and boundary scalar fields. Associating the bulk fields with dilatons and the boundary fields with glueballs of the confining gauge theory we also found the same high energy QCD scaling. Here, using this holographic mapping, we give a simple estimate for the mass ratios of the glueballs assuming the AdS slice approximation to be valid at low energies. We also compare these results to those coming from supergravity and lattice QCD.

Recently Polchinski and Strassler reproduced important observed properties of QCD from string theory in AdS space [1,2]. In these articles they used a model for the dual of a confining gauge theory which is approximately an AdS slice with an infrared cut-off. Using this model they were able [1] to obtain the high energy scaling of the QCD scattering amplitudes for fixed angles [3,4] as well as the Regge regime. Further they proposed [2] a way of analyzing the deep inelastic scattering and Bjorken scaling in terms of string theory. Using the same kind of AdS slice we proposed [5] a one to one holographic mapping between low energy string dilaton states in AdS bulk and massive composite operators on its boundary. From this mapping we also obtained a scaling for high energy amplitudes at fixed angles similar to that of QCD and of Polchinski and Strassler (see also [6–8]).

The gauge/string duality considered in [1,2] was inspired by the AdS/CFT correspondence proposed recently by Maldacena [9] where $SU(N)$ conformal gauge theory with $\mathcal{N} = 4$ supersymmetry is dual to string theory in AdS space (times a compact manifold). The prescriptions for realizing this correspondence obtaining boundary correlation functions in terms of bulk fields were proposed in [10,11] (see also [12] for a review). The AdS/CFT correspondence can be understood as a realization of the holographic principle [13–15].[1] This principle asserts that the degrees of freedom of a theory with gravity defined in a given space can be mapped on the corresponding boundary.

In the AdS/CFT correspondence the higher the energy of a given boundary process, the closer to the horizon is the bulk dual. Restricting boundary process to energies higher than some IR cut-off would then correspond to restricting the bulk to some region in the neighborhood of the horizon. That is a motivation for taking an AdS slice as an approximation for the space dual to a boundary confining gauge theory. Such a gauge theory with an infrared cut-off can be related to $\mathcal{N} = 1^*$ supersymmetric Yang–Mills theory [1,2] (see also [17]). This model leads to QCD-like behavior at high energies. An AdS slice was used before in [18,19] to propose a solution to the hierarchy problem. An approach based on string theory to $\mathcal{N} = 1$ super Yang–Mills has been proposed in [20].

An approach to QCD from the AdS/CFT correspondence was proposed by Witten [21]. It consists of breaking supersymmetry with different compactifications of AdS space. This involves at least one circle S_1 where anti-periodic boundary conditions are assumed for the fermionic fields (the bosonic ones are periodic). These compactifications leads to AdS–Schwarzschild black hole metrics that can be related to QCD_3 or QCD_4 [21]. This approach can be used to estimate glueball masses from supergravity models relating glueballs with the bulk dilaton modes in different dimensions [22]. This idea was implemented in [23] for QCD_3 and QCD_4 where the supergravity equations with the black hole metric do not allow one to obtain analytic solutions, but the eigenvalues related to the glueball masses can be found using a WKB method (see also [24–29]).

Here we use the mapping proposed in [5] between bulk dilatons and massive boundary operators defined in the AdS slice to estimate in a simple way the ratio of the boundary masses. This slice has an infrared cut-off that Polchinski and Strassler identified with the mass of the

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[1] For a covariant generalization of the holographic principle, see [16].

lightest glueball. Using this identification and assuming that the approximated duality is still valid for low energies we identify our boundary operators with glueballs of the confining gauge theory.

The general structure of the holographic mapping was taken from [30] where we introduced a mapping between scalar fields in AdS bulk and boundary. Using this mapping and the low energy string theory approximation we found the QCD like scaling for high energy amplitudes [5].

We consider an $\mathrm{AdS}_5 \times \mathrm{S}^5$ space with radius R described by Poincaré coordinates

$$\mathrm{d}s^2 = \frac{R^2}{z^2}\Big(\mathrm{d}z^2 + (\mathrm{d}\mathbf{x})^2 - \mathrm{d}t^2\Big) + R^2 \mathrm{d}\Omega_5^2 \,, \qquad (1)$$

where Ω_5 corresponds to the five dimensional sphere S^5. According to the AdS/CFT correspondence, glueballs are related to closed strings. At energies much lower than the string scale $1/\sqrt{\alpha'}$ string theory can be approximated by supergravity, where dilatons and gravitons play an important role [31]. In particular we will be interested in dilatons which are the string duals to scalar glueballs. We consider the dilaton to be in the s-wave state, so we will not take into account variations with respect to S^5 coordinates. The free dilaton field in AdS_5 with size $z_{\max}$ can be cast into the form [32]

$$\Phi(z,\mathbf{x},t) = \sum_{p=1}^{\infty} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{z^2\, J_2(u_p z)}{z_{\max}\; w_p(\mathbf{k})\, J_3(u_p z_{\max})} \times \Big\{ \mathbf{a}_p(\mathbf{k})\; \mathrm{e}^{-\mathrm{i} w_p(\mathbf{k}) t + \mathrm{i}\mathbf{k}\cdot\mathbf{x}} + \text{ h.c.} \Big\} \,, \qquad (2)$$

with $0 \le z \le z_{\max}$ and $w_p(\mathbf{k}) = \sqrt{u_p^2 + \mathbf{k}^2}$, h.c. means hermitean conjugate and the u_p are defined by

$$u_p z_{\max} = \chi_{_{2\,,\,p}} \qquad (3)$$

such that the Bessel function satisfies $J_2(\chi_{_{2\,,\,p}}) = 0$.

The operators $\mathbf{a}_p$, $\mathbf{a}_p^\dagger$ satisfy the commutation relations

$$\Big[\mathbf{a}_p(\mathbf{k}) \,,\, \mathbf{a}^\dagger_{p'}(\mathbf{k}') \Big] = 2\,(2\pi)^3 w_p(\mathbf{k}) \delta_{p\,p'}\, \delta^3(\mathbf{k} - \mathbf{k}') \,. \qquad (4)$$

On the boundary $(z = 0)$ of the AdS slice we consider massive composite operators $\Theta_i(\mathbf{x},t)$ representing glueballs with masses μ_i. The algebra of the corresponding creation–annihilation operators can be written as

$$\Big[\mathbf{b}_i(\mathbf{K}) \,,\, \mathbf{b}_i^\dagger(\mathbf{K}') \Big] = 2(2\pi)^3\, w_i(\mathbf{K})\, \delta^3(\mathbf{K} - \mathbf{K}') \,, \qquad (5)$$

where $w_i(\mathbf{K}) = \sqrt{\mathbf{K}^2 + \mu_i^2}$.

In the previous work [5] we considered a single glueball operator $\Theta(\mathbf{x},t)$. We have seen, following the general ideas of [30], that the discretization introduced by considering a slice of AdS makes it possible to establish a one to one mapping between bulk $(\mathbf{k}, u_p)$ and boundary $(\mathbf{K})$ momenta. We assume a trivial mapping between the angular parts of $\mathbf{k}$ and $\mathbf{K}$. Then in order to find a relation between $k = |\mathbf{k}|$, u_p and $K = |\mathbf{K}|$ we introduce a sequence of energy scales $\mathcal{E}_j$. Defining K_j to be a momentum in the interval $\mathcal{E}_{j-1} \le K \le \mathcal{E}_j$ we map the momentum space operator $\tilde{\Theta}(K_j)$ in a one to one relation with the dilaton operator of momentum k, u_p:

$$\tilde{\Theta}(K_j) \leftrightarrow \tilde{\Phi}(k, u_p) \,.$$

This way all the dilaton states are mapped into a single field on the boundary. Note that each interval $\mathcal{E}_{j-1} \le K \le \mathcal{E}_j$ is mapped into the entire range of k corresponding to a fixed u_p. For each positive integer j we choose a different p.

As we are looking at physical processes in the boundary theory which take place in a given energy range we can take $\mathcal{E}_1$ large enough so that the first energy interval $0 \le K \le \mathcal{E}_1 \equiv \mathcal{E}$ contains all the relevant physics. Then only one interval for K is necessary. In this case the above mapping reduces to

$$\tilde{\Theta}(K_1) \leftrightarrow \tilde{\Phi}(k, u_p) \quad (p \text{ fixed}).$$

For simplicity we took $p = 1$ in the previous work.

Here we want to describe physical processes involving a set of glueball operators $\Theta_i(\mathbf{x},t)$ $(i = 1,2,\dots)$ using the same kind of mapping. If we again introduce momentum operators $\tilde{\Theta}_i(K_j)$ with momentum K_j in the interval $\mathcal{E}_{j-1} \le K \le \mathcal{E}_j$ they would not be mapped in a one to one relation with bulk operators $\tilde{\Phi}(k, u_p)$ unless j is limited, since i and p are unlimited. Then such a mapping is possible if we introduce a restriction on the index j. The simplest choice is to take just one value for j. This is obtained taking $\mathcal{E}_1 \equiv \mathcal{E}$ large enough, which means that now $j = 1$. This recovers the previous solution and in this case the one to one mapping reads

$$\tilde{\Theta}_i(K) \leftrightarrow \tilde{\Phi}(k, u_i),$$

where we have dropped the index of K_1 since it is the only relevant boundary momentum.

This mapping can be written explicitly in terms of bulk and boundary creation–annihilation operators. We will impose the same relation proposed in [5]:

$$k\, \mathbf{a}_i(\mathbf{k}) = K\, \mathbf{b}_i(\mathbf{K}) \,, \\ k\, \mathbf{a}_i^\dagger(\mathbf{k}) = K\, \mathbf{b}_i^\dagger(\mathbf{K}) \,. \qquad (6)$$

For a general relation between bulk and boundary creation–annihilation operators in AdS_{n+1} see [30].

Requiring that the equations (6) preserve the canonical commutation relations (4) and (5) one finds that the moduli of the momenta are related, for each bulk and boundary state, by

$$k = \frac{u_i}{2} \left[\frac{\mathcal{E} + \sqrt{\mathcal{E}^2 + \mu_i^2}}{K + \sqrt{K^2 + \mu_i^2}} - \frac{K + \sqrt{K^2 + \mu_i^2}}{\mathcal{E} + \sqrt{\mathcal{E}^2 + \mu_i^2}} \right] , \qquad (7)$$

where $0 \le K \le \mathcal{E}$. Note that this mapping contains as a particular case the previous one [5] which can be reobtained here if we consider the masses μ_i to be completely degenerated.

Now we associate the size $z_{\max}$ of the AdS space with the mass of the lightest glueball which we choose to be μ_1:

$$z_{\max} = \frac{\chi_{2\,,\,1}}{\mu_1}\,, \tag{8}$$

so that from equation (3) we have

$$u_i = \frac{\chi_{2\,,\,i}}{\chi_{2\,,\,1}}\,\mu_1. \tag{9}$$

An approximate expression for the mapping (7) can be obtained choosing appropriate energy scales. We take $\mathcal{E}$ to be the string scale $1/\sqrt{\alpha'}$ assuming that $K \ll \mathcal{E}$. On the other side we restrict the momenta K associated with glueballs to be much larger than their masses μ_i. Then we have

$$\mu_i \ll K \ll \frac{1}{\sqrt{\alpha'}}. \tag{10}$$

In this regime the mapping (7) reduces to

$$k \approx \frac{u_i}{2\sqrt{\alpha'}K}\,. \tag{11}$$

This approximate mapping gives a high energy scaling similar to QCD [5]. Using the conditions (10) together with the above mapping we see that the bulk momenta satisfy

$$u_i \ll k \ll \left(\frac{u_i}{\mu_i}\right)\frac{1}{\sqrt{\alpha'}}. \tag{12}$$

Note that the supergravity approximation holds for $k \ll 1/\sqrt{\alpha'}$. So in order to keep this approximation valid for all glueball operators Θ_i the factor u_i/μ_i should be nearly constant. We then impose that

$$\frac{u_i}{\mu_i} = \text{constant}\,. \tag{13}$$

So the glueball masses are related to the zeros of the Bessel functions by

$$\frac{\mu_i}{\mu_1} = \frac{\chi_{2\,,\,i}}{\chi_{2\,,\,1}}\,. \tag{14}$$

Using the values of these zeros one finds the ratio of the glueball masses for the state 0^{++} and its excitations. We are using the conventional notation for these states with spin zero and positive parity and charge conjugation. In order to compare our results from bulk/boundary holographic mapping with those coming from lattice we adopt the mass of the first state as an input. Our results are in good agreement with lattice [33,34] and AdS–Schwarzschild black hole supergravity calculations as seen in Table 1. It is interesting to mention that an approach to estimate glueball masses in Yang–Mills* from a deformed AdS space was discussed very recently in [35].

We can generalize the above results to AdS$_{n+1}$. In this case massless bulk fields are expanded in terms of the Bessel functions $J_{n/2}$ and the mass ratios for the n dimensional "glueballs" are given in terms of their zeros. In particular for AdS$_4$, where one expects to recover results from QCD$_3$, we find

$$\frac{\mu_i}{\mu_1} = \frac{\chi_{3/2\,,\,i}}{\chi_{3/2\,,\,1}}\,. \tag{15}$$

Using this relation we obtain the ratio of masses presented in Table 2 together with lattice and AdS–Schwarzschild black hole supergravity calculations. The agreement here is also good.

Table 1. Masses of the first few 0^{++} glueballs for QCD$_4$ with $SU(N)$ and $N=3$, in GeV, from lattice [33,34], from AdS–Schwarzschild black hole supergravity [23] and our results from bulk/boundary holographic mapping, (14)

QCD$_4$ state	Lattice, $N=3$	Supergravity	Bulk/boundary
0^{++}	1.61 ± 0.15	1.61 (input)	1.61 (input)
0^{++*}	2.8	2.38	2.64
0^{++**}	–	3.11	3.64
0^{++***}	–	3.82	4.64
0^{++****}	–	4.52	5.63
$0^{++*****}$	–	5.21	6.62

It is interesting to see if the AdS slice considered here can be related to the AdS–Schwarzschild black hole metric proposed by Witten [21]. Witten's proposal for the case of QCD$_3$ corresponds to the ten dimensional metric [23]

$$\mathrm{d}s^2 = R^2\left(\rho^2 - \frac{b^4}{\rho^2}\right)^{-1}\mathrm{d}\rho^2 + R^2\left(\rho^2 - \frac{b^4}{\rho^2}\right)\mathrm{d}\tau^2 + R^2\rho^2(\mathrm{d}\mathbf{x})^2 + R^2\mathrm{d}\Omega_5^2\,, \tag{16}$$

where $\rho \geq b\,,\ R^2 = l_s^2\sqrt{4\pi g_s N}$, and b is inversely proportional to the compactification radius of S$_1$ where the τ variable is defined.

If we qualitatively neglect the τ contribution to the metric in the limit of very little compactification radius and then take the limit $\rho \gg b$ this metric is approximated by

$$\mathrm{d}s^2 = \frac{R^2}{\rho^2}\mathrm{d}\rho^2 + R^2\rho^2\mathrm{d}\tau^2 + R^2\rho^2(\mathrm{d}\mathbf{x})^2 + R^2\mathrm{d}\Omega_5^2\,. \tag{17}$$

That is an AdS$_4 \times S^5$ space that takes a form similar to (1) if we change the axial coordinate to $z = 1/\rho$. In Witten's framework one must impose regularity conditions at $\rho = b$ because of the presence of the horizon at this position. In our approximation in order to retain this physical condition we impose boundary conditions there and associate it to the cut of our slice ($b = 1/z_{\max}$). This AdS$_4$ slice is the one used to estimate the glueball mass ratios related to the three dimensional gauge theory (15). So we can think of our AdS$_4$ slice as a naive approximation to Witten's proposal.

An analogous situation could also be considered for Witten's proposal to QCD$_4$. In that case the situation is more involved because of the form of the metric coming from the compactification of AdS$_7 \times$ S^4.

Table 2. 0^{++} glueball masses for QCD$_3$ with $SU(N)$ from lattice [33,34](in units of string tension), from AdS–Schwarzschild black hole supergravity [23] and our results from bulk/boundary holographic mapping, (15)

QCD$_3$ state	Lattice, $N = 3$	Lattice, $N \to \infty$	Supergravity	Bulk/boundary
0^{++}	4.329 ± 0.041	4.065 ± 0.055	4.07 (input)	4.07 (input)
0^{++*}	6.52 ± 0.09	6.18 ± 0.13	7.02	7.00
0^{++**}	8.23 ± 0.17	7.99 ± 0.22	9.92	9.88
0^{++***}	–	–	12.80	12.74
0^{++****}	–	–	15.67	15.60
$0^{++*****}$	–	–	18.54	18.45

In conclusion we have seen that the bulk/boundary holographic mapping which reproduces the high energy scaling of QCD like theories can also be applied to estimate glueball mass ratios. We hope that this mapping can be used to describe other particle states that may be related to some properties of QCD.

It is important to remark that one can obtain a similar result for the ratio of the glueball masses considering other mappings between bulk and boundary creation–annihilation operators instead of (6). For example one could take $\mathbf{a}_i = \mathbf{b}_i$. This would contain the solution $k = K$ implying that the masses of the glueballs are identically equal to the values of the axial bulk momenta u_i. However such a trivial mapping does not seem to reproduce the high energy QCD scaling.

Let us mention that we used a solution for the dilaton field corresponding to Dirichlet boundary conditions at $z = 0$ and $z = z_{\max}$. This allows for the existence of Bessel functions but not the divergent Neumann solutions. Other boundary conditions can also be considered in the same context.

We have also obtained elsewhere [36] these mass ratios for scalar glueballs starting with the same AdS slice as discussed here without using the holographic mapping of [5] but assuming the stronger condition of relating directly the dilaton modes with the glueball masses. The consistency between these results seems to indicate that the holographic mapping found before may indeed be valid within the approximations and the energy region considered.

What is surprising in this bulk/boundary holographic mapping is that it seems to describe features of both high and low energy regimes of the boundary theory, since it gives information about the high energy scaling and mass spectrum.

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